Exercices d'entraînements

Calculer les dérivées des fonctions données:

1°) $f(x) = 2x^2 - 3x + 5$

2°) $f(x) = -5x^3 + \frac{x^2}{2} - x - \sqrt{2}$

3°) $f(x) = \frac{1}{x^2 - 2x + 3}$

4°) $f(x) = \frac{3}{x} + x$

5°) $f(x) = \frac{3}{x} + \frac{1}{x^2} - 2$

6°) $f(x) = (2x-1)(x+5)$

7°) $f(x) = \frac{2x-1}{x+5}$

8°) $f(x) = \frac{x^2+x+1}{3-x}$

9°) $f(x) = \frac{x^2-5x}{x^2+x-2}$

10°) $f(x) = (4x-1)^2$

11°) $f(x) = (1-2x)^3$

12°) $f(x) = \frac{1}{(4x-1)^2}$

13°) $f(x) = \cos(3x+4)$

14°) $f(x) = \frac{\cos(2x)}{\sin(3x)}$

Solutions

1°) $f(x) = 2x^2 - 3x + 5$

$f'(x) = 2.2x - 3 + 0$

Donc $\boxed{f'(x) = 4x - 3}$

2°) $f(x) = -5x^3 + \frac{1}{2}x^2 - x - \sqrt{2}$

$f'(x) = -5\times 3x^2 + \frac{1}{2}.2x - 1 - 0$

Donc $\boxed{f'(x) = -15x^2 + x - 1}$

3°) $f(x) = \frac{1}{x^2-2x+3} = \frac{1}{v}$ avec $\begin{cases} v = x^2 - 2x + 3 \\ v' = 2x - 2 \end{cases}$

$\boxed{f'(x) = -\frac{v'}{v^2} = -\frac{2x-2}{(x^2-2x+3)^2}}$

4°) $f(x) = 3.\frac{1}{x} + x$

Donc $f'(x) = 3.\left(-\frac{1}{x^2}\right) + 1$

Donc $\boxed{f'(x) = \frac{-3}{x^2} + 1}$

5°) $f(x) = \dfrac{3}{x} + x^{-2} - 2$

Donc $f'(x) = 3 \times \left(-\dfrac{1}{x^2}\right) + (-2)x^{-3} - 0$

Donc $\boxed{f'(x) = -\dfrac{3}{x^2} - \dfrac{2}{x^3}}$

6°) $f(x) = (2x-1)(x+5) = u.v$

Donc $f'(x) = u'v + uv' = 2(x+5) + (2x-1) \times 1$

$= 2x + 10 + 2x - 1$

Donc $\boxed{f'(x) = 4x + 9}$

7°) $f(x) = \dfrac{2x-1}{x+5} = \dfrac{u}{v}$ avec $\begin{cases} u = 2x-1 & u' = 2 \\ v = x+5 & v' = 1 \end{cases}$

Donc $f'(x) = \dfrac{u'v - uv'}{v^2} = \dfrac{2(x+5) - (2x-1) \times 1}{(x+5)^2}$

Donc $\boxed{f'(x) = \dfrac{11}{(x+5)^2}}$

8°) $f(x) = \dfrac{x^2 + x + 1}{3 - x} = \dfrac{u}{v}$

avec $\begin{cases} u = x^2 + x + 1 & u' = 2x + 1 \\ v = 3 - x & v' = -1 \end{cases}$

$f'(x) = \dfrac{u'v - uv'}{v^2} = \dfrac{(2x+1)(3-x) - (x^2+x+1) \times (-1)}{(3-x)^2}$

$f'(x) = \dfrac{6x - 2x^2 + 3 - x + x^2 + x + 1}{(3-x)^2}$

D'où $\boxed{f'(x) = \dfrac{-x^2 + 6x + 4}{(3-x)^2}}$

9°) $f(x) = \dfrac{x^2 - 5x}{x^2 + x - 2} = \dfrac{u}{v}$ avec $\begin{cases} u = x^2 - 5x & u' = 2x - 5 \\ v = x^2 + x - 2 & v' = 2x + 1 \end{cases}$

Donc

$f'(x) = \dfrac{u'v - uv'}{v^2} = \dfrac{(2x-5)(x^2+x-2) - (x^2-5x)(2x+1)}{(x^2+x-2)^2}$

$f'(x) = \dfrac{2x^3 + 2x^2 - 4x - 5x^2 - 5x + 10 - (2x^3 + x^2 - 10x^2 - 5x)}{(x^2+x-2)^2}$

D'où $\boxed{f'(x) = \dfrac{6x^2 - 4x + 10}{(x^2+x-2)^2}}$

10°/ $f(x) = (4x-1)^2$

$f'(x) = 2.(4x-1)^1 \times 4$

Donc $\boxed{f'(x) = 8.(4x-1)}$

11°/ $f(x) = (1-2x)^3$

$f'(x) = 3.(1-2x)^2 \times (-2)$

Donc $\boxed{f'(x) = -6.(1-2x)^2}$

12°/ $f(x) = \dfrac{1}{(4x-1)^2} = (4x-1)^{-2}$

Donc $f'(x) = -2(4x-1)^{-3} \times 4$

D'où $\boxed{f'(x) = \dfrac{-8}{(4x-1)^3}}$

13°) $f(x) = \cos(3x+4)$

$f'(x) = -\sin(3x+4) \times 3$

Donc $\boxed{f'(x) = -3\sin(3x+4)}$

14°) $f(x) = \dfrac{\cos(2x)}{\sin(3x)} = \dfrac{u}{v}$

avec $\begin{cases} u = \cos(2x) & u' = -2\sin(2x) \\ v = \sin(3x) & v' = 3\cos(3x) \end{cases}$

$f'(x) = \dfrac{u'v - uv'}{v^2} = \dfrac{-2\sin(2x).\sin(3x) - 3\cos(2x)\cos(3x)}{\sin^2(3x)}$

Donc :

$$\boxed{f'(x) = \dfrac{-2\sin(2x).\sin(3x) - 3\cos(2x).\cos(3x)}{\sin^2(3x)}}$$